Predictions for lepton flavor violation in Z decays in supersymmetry with left-right symmetry

M. Frank*
Department of Physics, Concordia University, 1455 De Maisonneuve Boulevard West, Montreal, Quebec, Canada H3G 1M8

(Received 13 August 2001; published 11 January 2002)

Recent experimental results on neutrino oscillations and the muon anomalous magnetic moment, combined with radiative lepton decays, set strict limits on fractional slepton and sneutrino flavor violating mixings. We study the effect of these mixings on Z lepton flavor violating decays and the implications for the observability of these decays in view of the new GigaZ option at the e^+e^- collider, expected to reach a sensitivity of up to 10^{-8} for branching rates. The largest branching ratio predicted is of order 10^{-11}–10^{-12} for the decays $Z \to \tau^\pm \mu^\mp$, therefore precluding observation of a signal at DESY TESLA. The branching ratios for $Z \to \tau^\pm e^\mp$ and $Z \to e^\pm \mu^\mp$ are expected to be even smaller.

DOI: 10.1103/PhysRevD.65.033011 PACS number(s): 13.35.Dx, 12.60.Jv, 14.60.Fg

I. INTRODUCTION

High precision measurements of the low energy processes can provide extremely useful probes of physics beyond the standard model. If the recent measurement of the 2.6σ deviation of the anomalous magnetic moment of the muon from the expected value in the standard model [1] is interpreted as arising from supersymmetry, bounds can be obtained on the fractional flavor splitting in the slepton mass matrix or on the phase of the dipole operator arising in electric dipole moments. The elements entering such analyses are precise constraints on lepton flavor violating (LFV) decays, such as $\mu \to e\gamma, \tau \to \mu\gamma$, $\mu - e$ conversion, and the electric dipole moment of the electron. Assuming slepton universality, lepton dipole operators for different generations are expected to be related by ratios of lepton masses. However, if universality is broken, these ratios are modifed by slepton flavor violation in a model-dependent manner. Such analyses have been carried out in the minimal supersymmetric model [2] and a supersymmetric model with left-right symmetry [3]. The size of the slepton flavor violating masses in supersymmetry depends on the symmetry of the model. For example, in the minimal supersymmetric standard model (MSSM) mass splittings of the left-handed sleptons are thought to dominate mass splittings of the right-handed fermions, which are assumed negligible. In models with left-right symmetry, such as the left-right supersymmetric model (LRSUSY) and SO(10), the fractional splitting of the left- and right-handed sleptons is of the same order of magnitude [3,4]. In addition, new contributions to the muon anomalous magnetic moment [5] can modify considerably the relationship between lepton flavor conserving and lepton flavor violating dipole operators [3].

Motivated by the interest in lepton number violating processes, we apply the constraints from radiative lepton decays, $\mu - e$ conversion and the anomalous magnetic moment of the muon to the lepton flavor violating decays of the Z. The best direct limits on the branching ratios are weak compared to $\mu \to e\gamma$, $\mu - e$ conversion and $\mu \to 3e$ decays:

$$BR(Z \to e^\pm \mu^\mp) < 1.7 \times 10^{-6} \quad [6] \tag{1}$$

$$BR(Z \to e^\pm \tau^\mp) < 9.8 \times 10^{-6} \quad [6,7] \tag{2}$$

$$BR(Z \to \mu^\pm \tau^\mp) < 1.2 \times 10^{-5} \quad [6,8]. \tag{3}$$

However, these branching ratios are projected to improve in the GigaZ option of the DESY TeV Energy Superconducting Linear Accelerater (TESLA) to a sensitivity of about 10^{-8} [9]. Older studies of studies of LFV decays of the Z in supersymmetry exist [10,11], as well as recent studies of the context of massive neutrinos [12] which incorporate data from neutrino oscillations [13]. However, no predictions exist in a supersymmetric model which incorporates the constraints on slepton mixing from other LFV decays. In this work we update our previous work on the lepton favor violating decays of the Z boson [14] in the context of a left-right supersymmetric model. The LRSUSY model incorporates neutrino masses and oscillations naturally through the seesaw mechanism, while providing a supersymmetric explanation for the observed deviation of the anomalous magnetic moment of the muon from the standard model value. The analysis presented here connects Z lepton flavor violating decays to other lepton flavor violating decays, incorporating constraints on mixings from neutrino oscillations and $(g-2)_\mu$. Although this investigation has some model-dependent features, it has the advantage that definte predictions are obtained, which, coupled to a measurement of the branching ratios, can confirm or rule out such a model.

II. THE LEFT-RIGHT SUPERSYMMETRIC MODEL

The LRSUSY model, based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, has matter doublets for both left- and right-handed fermions and the corresponding left- and right-handed scalar partners (sleptons and squarks) [15]. In the gauge sector, corresponding to $SU(2)_L$ and $SU(2)_R$, there are triplet gauge bosons $(W^{+,-},W^0)_L$, $(W^{+,-},W^0)_R$ and a singlet gauge boson V corresponding to $U(1)_{B-L}$, together with their superpartners. The Higgs sector of this model consists of two Higgs bidoublets, $\Phi_u(\frac{1}{2},\frac{1}{2},0)$ and $\Phi_d(\frac{1}{2},\frac{1}{2},0)$, which are required to give masses to both the up and down quarks. In addition, the spontaneous symmetry breaking of

*Email address: mfrank@vax2.concordia.ca

the group $SU(2)_R\times U(1)_{B-L}$ to the hypercharge symmetry group $U(1)_Y$ is accomplished by introducing the Higgs triplet fields $\Delta_L(1,0,2)$ and $\Delta_R(0,1,2)$. With this choice a large Majorana mass can be generated for the right-handed neutrino and a small one for the left-handed neutrino [16]. In addition to the triplets $\Delta_{L,R}$, the model must contain two additional triplets $\delta_L(1,0,-2)$ and $\delta_R(0,1,-2)$, with quantum number $B-L=-2$, to ensure cancellation of the anomalies in the fermionic sector.

The vacuum expectation values (VEV's) of the Higgs fields needed to break the symmetry to the standard model are taken, for simplification: $\langle\Delta_L\rangle=0$ and

$$\langle\Delta_R\rangle=\begin{pmatrix}0 & 0\\ v_R & 0\end{pmatrix},\quad \langle\Phi_u\rangle=\begin{pmatrix}\kappa_u & 0\\ 0 & 0\end{pmatrix}$$

and

$$\langle\Phi_d\rangle=\begin{pmatrix}0 & 0\\ 0 & \kappa_d\end{pmatrix}.$$

In the supersymmetric sector of the model there are six singly charged charginos,

$$\Psi^{+}=(-i\lambda_L^{+},-i\lambda_R^{+},\tilde{\Phi}_u^{+},\tilde{\Phi}_d^{+},\tilde{\delta}_L^{+},\tilde{\delta}_R^{+})$$

and

$$\Psi^{-}=(-i\lambda_L^{-},-i\lambda_R^{-},\tilde{\Phi}_u^{-},\tilde{\Phi}_d^{-},\tilde{\Delta}_L^{-},\tilde{\Delta}_R^{-}); \qquad (4)$$

two doubly charged Higgsinos,

$$\Psi^{++}=(\tilde{\delta}_L^{++},\tilde{\delta}_R^{++})\quad\text{and}\quad\Psi^{--}=(\tilde{\Delta}_L^{--},\tilde{\Delta}_R^{--}); \qquad (5)$$

and nine neutralinos,

$$\Psi^{0}=(-i\lambda_B^{0},-i\lambda_L^{0},-i\lambda_R^{0},\tilde{\Phi}_u^{0},\tilde{\Phi}_d^{0},\tilde{\Delta}_L^{0},\tilde{\delta}_L^{0},\tilde{\Delta}_R^{0},\tilde{\delta}_R^{0}). \qquad (6)$$

The sources of flavor violation in the LRSUSY model come from either the Yukawa potential or the trilinear scalar coupling. The interaction of fermions with scalar (Higgs) fields has the following form:

$$\mathcal{L}_Y=\mathbf{h}_u\overline{Q_L}\Phi_uQ_R+\mathbf{h}_d\overline{Q_L}\Phi_dQ_R+\mathbf{h}_\nu\overline{L_L}\Phi_uL_R+\mathbf{h}_e\overline{L_L}\Phi_dL_R+\text{H.c.}$$

$$\mathcal{L}_M=i\mathbf{h}_{LR}(L_L^TC^{-1}\tau_2\Delta_LL_L+L_R^TC^{-1}\tau_2\Delta_RL_R)+\text{H.c.} \qquad (7)$$

where $\mathbf{h}_u$, $\mathbf{h}_d$, $\mathbf{h}_\nu$ and $\mathbf{h}_e$ are the Yukawa couplings for the up and down quarks and neutrino and electron, respectively, and $\mathbf{h}_{LR}$ is the coupling for the triplet Higgs bosons. In addition, soft supersymmetry breaking terms which generate masses for the charged slepton fields also induce lepton flavor violation. The SUSY-breaking term for the Higgs bosons and lepton sector in LRSUSY is given by

$$\begin{aligned}\mathcal{L}_{soft}=&-[\mathbf{A}_l^i\mathbf{h}_l^{(i)}\tilde{L}^T\tau_2\Phi_i\tau_2\tilde{L}^c+i\mathbf{A}_{LR}\mathbf{h}_{LR}\\&\times(\tilde{L}^T\tau_2\Delta_L\tilde{L}+L^{cT}\tau_2\Delta_R\tilde{L}^c)+m_\Phi^{(ij)2}\Phi_i^\dagger\Phi_j]\\&+[(m_L^2)_{ij}\tilde{l}_{Li}^\dagger\tilde{l}_{Lj}+(m_R^2)_{ij}\tilde{l}_{Ri}^\dagger\tilde{l}_{Rj}+(M_N^2)_{ij}\tilde{N}_i^*\tilde{N}_j^*]\\&+M_{LR}^2[Tr(\Delta_R\delta_R)+Tr(\Delta_L\delta_L)]+B\mu_{ij}\Phi_i\Phi_j\end{aligned} \qquad (8)$$

where the **A** matrices (A_u, A_d, A_ν and A_e) are of similar form to the Yukawa couplings and provide additional sources of flavor violation, B is a mass term and $\tilde{N}$ is the scalar component of the right-handed neutrino supermultiplet. The inter-generational slepton mixing ($\tilde{e}$, $\tilde{\mu}$ and $\tilde{\tau}$) and also left-right slepton mixing ($\tilde{e}_L,\tilde{e}_R,\tilde{\mu}_L,\tilde{\mu}_R,\tilde{\tau}_L,\tilde{\tau}_R$) cause the off-diagonal nature of the matrices, and therefore are responsible for flavor violation. The slepton mass matrix is, incorporating some elements of the left-right symmetry [17]:

$$(m_{\tilde{l}}^2)_{eff}=\begin{pmatrix}\tilde{m}_L^2+D_L & (\tilde{m}_L)^2_{21} & (\tilde{m}_L)^2_{31} & \mathcal{A}_e & 0 & 0\\ (\tilde{m}_L)^2_{21} & \tilde{m}_L^2+D_L & (\tilde{m}_L)^2_{32} & 0 & \mathcal{A}_\mu & 0\\ (\tilde{m}_L)^2_{31} & (\tilde{m}_L)^2_{32} & \tilde{m}_L^2+D_L & 0 & 0 & \mathcal{A}_\tau\\ \mathcal{A}_e & 0 & 0 & \tilde{m}_R^2+D_R & (\tilde{m}_R)^2_{21} & (\tilde{m}_R)^2_{31}\\ 0 & \mathcal{A}_\mu & 0 & (\tilde{m}_R)^2_{21} & \tilde{m}_R^2+D_R & (\tilde{m}_R)^2_{32}\\ 0 & 0 & \mathcal{A}_\tau & (\tilde{m}_R)^2_{31} & (\tilde{m}_R)^2_{32} & \tilde{m}_R^2+D_R\end{pmatrix}, \qquad (9)$$

where

$$\mathcal{A}_l=A_lm_l+m_l\mu\tan\beta,\quad (l=e,\mu,\tau),$$

$$D_L=M_{Z_L}^2(T_3/2+\sin^2\theta_W)\cos2\beta+M_{Z_R}^2\sin^2\theta_W/\sin2\theta_W\sin2\beta,$$

and

$$D_R=-M_{Z_L}^2\sin^2\theta_W\cos2\beta+M_{Z_R}^2(T_3/2-\sin^2\theta_W/\cos2\theta_W)\times\sin2\beta.$$

From the point of view of Z lepton flavor violating decays, only the left-left and right-right mixing will be important, and we can approximate the slepton mixing matrix as block diagonal, expressed as $S^{L,R}$

$$\tilde{l}_{\alpha L,R}=S^{L,R}_{\alpha i}\tilde{l}^{L,R}_{i} \qquad (10)$$

with $\alpha=e,\mu,\tau$ and $i=1,2,3$. This approximation allows the left-right mixings to be proportional to the values of the trilinear parameters $\mathcal{A}_l$.

The full mass for left- and right-handed sneutrino has, in general, a complicated 12×12 matrix structure, but is possible to construct an effective 6×6 matrix for the light sneutrinos using the seesaw mechanism [18]:

$$(m^2_{\tilde{\nu}})_{eff}=\begin{pmatrix} m_L^2-\mathcal{A}'_\nu(A_\nu-2A_N)(m_DM^{-2}m_D^\dagger) & \mathcal{A}'^{*}_\nu(m_DM^{-1}m_D^\dagger) \\ \mathcal{A}'_\nu(m_DM^{-1}m_D^\dagger)^* & m_L^2-\mathcal{A}'_\nu(A_\nu-2A_N)(m_DM^{-2}m_D^\dagger)\end{pmatrix} \qquad (11)$$

where $\mathcal{A}'_\nu\sim 2A_\nu+A_N+2\mu\cot\beta$. In the LRSUSY model, the left-handed neutrino mass is allowed to be nonzero, but can be made small through the seesaw mechanism, as long as the right-handed neutrino is very heavy (masses of order 10^{10} TeV or so are consistent with the 1 eV limit on the left-handed neutrino mass). Despite the presence of the two scalar neutrinos, the mixing between the right-handed and the left-handed sneutrinos is small, due to the seesaw mechanism in the sfermion sector: the left-right elements of the sneutrino mass matrix are proportional to the Dirac neutrino mass, but the right-right element of the sneutrino mass matrix is very heavy, so the mixing of sneutrino will be supressed by the inverse M_N^2, the right-handed neutrino mass. We denote the light sneutrino mixing matrix by K and express their mixing as

$$\tilde{\nu}_\alpha=K_{\alpha i}\tilde{\nu}_i \qquad (12)$$

with $\alpha=\nu_e,\nu_\mu,\nu_\tau$ and $i=1,2,3$.

III. LEPTON FLAVOR VIOLATING DECAYS OF THE Z

The cross section for the process $Z\to\overline{l_1}l_2+l_1\overline{l_2}$ arises at the one-loop level through the decay amplitude

$$\mathcal{M}=-\frac{ig\alpha_W}{4\pi}\epsilon^\mu\overline{u}_{l_2}(p_2)\Gamma_\mu u_{l_1}(-p_1) \qquad (13)$$

with

$$\Gamma_\mu=\gamma_\mu(f_V-f_A\gamma_5)+\frac{q^\nu}{M_W}(if_M+f_E\gamma_5)\sigma_{\mu\nu} \qquad (14)$$

where f_V and f_A stand for the vector and axial vector couplings and f_M and f_E for the magnetic and electric dipole moments of the final fermions. For on-shell photons, the radiative decays of leptons are due exclusively to dipole transitions, while the LFV decays of the Z will depend in principle on all four form factors. These form factors are model-dependent: however, in the limit of massless external leptons there is only one independent form factor in each case, such that $f_M^\gamma=f_E^\gamma$ and $f_A^Z=f_V^Z$, $f_M^Z=f_E^Z=0$. The branching ratio for the lepton flavor violating Z decay becomes

$$BR(Z\to\overline{l_1}l_2+l_1\overline{l_2})=\frac{M_Z}{\Gamma_Z}[|\Gamma^L_{l_1l_2}|^2+|\Gamma^R_{l_1l_2}|^2], \qquad (15)$$

where the nonoblique functions $\Gamma^L_{l_1l_2}$ and $\Gamma^R_{l_1l_2}$ depend on the $V-A$ or $V+A$ character of the theory. We list below the supersymmetric contributions to the amplitude for the decay $Z\to\overline{l_1}l_2+l_1\overline{l_2}$, including the mixing of scalar leptons and scalar neutrinos and of the charginos and neutralinos in the model (and including the self-energy graphs). The contributions from the LRSUSY model to the decay $Z\to\overline{l_1}l_2+l_1\overline{l_2}$ fall into three categories: chargino-sneutrino, neutralino-slepton and doubly charged Higgsinos-sleptons [19].

The chargino-sneutrino contribution is

$$\Gamma^L_{l_1\overline{l}_2}(c)=\frac{-ig^3}{32\pi^2\cos\theta_W}\sum_{k=1}^{3}K_{\nu_1k}K_{\nu_2k}\Bigg\{\sum_{i=1}^{4}(|V_{i1}|^2)\tilde{\lambda}_{\nu_k}[C_{24}(\tilde{\lambda}_i^-,\tilde{\lambda}_{\nu_k},\tilde{\lambda}_{\nu_k})+B_1(0,\tilde{\lambda}_i^-,\tilde{\lambda}_{\nu_k})]$$
$$+2\sum_{i=1}^{4}\sum_{j=1}^{4}(V^*_{i1}V_{j1})\{O^{L'}_{ij}[2C_{24}(\tilde{\lambda}_{\nu_k},\tilde{\lambda}_i^-,\tilde{\lambda}_j^+)-\tfrac{1}{2}-\lambda_ZC_{23}(\tilde{\lambda}_{\nu_k},\tilde{\lambda}_i^-,\tilde{\lambda}_j^+)]-O^{R'}_{ij}\tilde{\lambda}^{\pm}_{ij}C_0(\tilde{\lambda}_{\nu_k},\tilde{\lambda}_i^-,\tilde{\lambda}_j^+)\}\Bigg\}. \qquad (16)$$

The neutralino-left slepton contribution is

$$\Gamma^{L}_{l_1\bar{l}_2}(n)=\frac{-ig^3}{64\pi^2\cos\theta_W}\sum_{k=1}^{3}S^L_{l_1k}S^L_{l_2k}\Bigg\{\sum_{i=1}^{4}(|\tan\theta_W N_{i1}+N_{i2}|^2)\cos 2\theta_W\tilde{\lambda}_{k_L}[C_{24}(\tilde{\lambda}^0_i,\tilde{\lambda}_{k_L},\tilde{\lambda}_{k_L})+\tfrac{1}{2}B_1(0,\tilde{\lambda}^0_i,\tilde{\lambda}_{k_L})]$$
$$+2\sum_{i=1}^{4}\sum_{j=1}^{4}(\tan\theta_W N^*_{i1}+N^*_{i2})(\tan\theta_W N_{j1}+N_{j2})\times\{O^{L''}_{ij}[2C_{24}(\tilde{\lambda}_{k_L},\tilde{\lambda}^0_i,\tilde{\lambda}^0_j)-\tfrac{1}{2}-\lambda_Z C_{23}(\tilde{\lambda}_{k_L},\tilde{\lambda}^0_i,\tilde{\lambda}^0_j)]$$
$$-O^{R''}_{ij}\tilde{\lambda}^0_{ij}C_0(\tilde{\lambda}_{k_L},\tilde{\lambda}^0_i,\tilde{\lambda}^0_j)\}\Bigg\}. \tag{17}$$

The neutralino-right slepton contribution is

$$\Gamma^{R}_{l_1\bar{l}_2}(n)=\frac{-ig^3}{64\pi^2\cos\theta_W}\sum_{k=1}^{3}S^R_{l_1k}S^R_{l_2k}\Bigg\{\sum_{i=1}^{4}(|-2\tan\theta_W N_{i1}+N_{i3}|^2)\cos 2\theta_W\tilde{\lambda}_{k_R}[C_{24}(\tilde{\lambda}^0_i,\tilde{\lambda}_{k_R},\tilde{\lambda}_{k_R})+\tfrac{1}{2}B_1(0,\tilde{\lambda}^0_i,\tilde{\lambda}_{k_R})]$$
$$+2\sum_{i=1}^{4}\sum_{j=1}^{4}(-2\tan\theta_W N^*_{i1}+N^*_{i3})(-2\tan\theta_W N_{j1}+N_{j3})\{O^{R''}_{ij}[2C_{24}(\tilde{\lambda}_{l_{k_R}},\tilde{\lambda}^0_i,\tilde{\lambda}^0_j)-\tfrac{1}{2}-\lambda_Z C_{23}(\tilde{\lambda}_{l_{k_R}},\tilde{\lambda}^0_i,\tilde{\lambda}^0_j)]$$
$$-O^{L''}_{ij}\tilde{\lambda}^0_{ij}C_0(\tilde{\lambda}_{l_{k_R}},\tilde{\lambda}^0_i,\tilde{\lambda}^0_j)\}\Bigg\} \tag{18}$$

and the doubly charged Higgsino contribution is

$$\Gamma^{R}_{l_1\bar{l}_2}(C)=\frac{-ig}{64\pi^2\cos\theta_W}\sum_{k=1}^{3}S^R_{l_1k}S^R_{l_2k}(h_{LR})_{l_1k}(h_{LR})_{l_2k}\sin^2\theta_W\{\tilde{\lambda}_{k_R}[C_{24}(\tilde{\lambda}^{--}_{\delta},\tilde{\lambda}_{k_R},\tilde{\lambda}_{k_R})-\tfrac{1}{2}B_1(0,\tilde{\lambda}_{\delta},\tilde{\lambda}_{k_R}]$$
$$+\tfrac{1}{2}[2C_{24}(\tilde{\lambda}_{k_R},\tilde{\lambda}_{\delta},\tilde{\lambda}_{\delta})-\tfrac{1}{2}-\lambda_Z C_{23}(\tilde{\lambda}_{k_R},\tilde{\lambda}_{\delta},\tilde{\lambda}_{\delta})]\} \tag{19}$$

where $\lambda_{k_{L,R}}=m^2_{\tilde{l}_{k(L,R)}}/M^2_{\chi^0}$, $\lambda_{\nu_k}=m^2_{\tilde{\nu}_k}/M^2_{\chi^-}$, $\lambda^{0,\pm}_{ij}=M_{\chi^{0,\pm}_i}/M_{\chi^{0,\pm}_j}$, $\lambda_Z=M^2_Z/M^2_{\chi^0}$; the functions $C_{ij}(\lambda_i,\lambda_j,\lambda_k)$ are the three-point functions, and $B_1(0,\lambda_i,\lambda_j)$ is a two-point function associated with the self-energy graphs. We follow here the notation and conventions of Ref. [20]. The matrix elements O'_{ij} and N_{ij} arise from neutralino interaction and mixing at the vertices; O_{ij} and V_{ij}, U_{ij} arise from chargino interaction and mixing at the vertices and are given by

$$O^L_{ij}=V^*_{i2}V_{j2}+\frac{1}{2}V^*_{i3}V_{j3}+\frac{1}{2}V^*_{i4}V_{j4}-\sin^2\theta_W\delta_{ij} \tag{20}$$

$$O^R_{ij}=U^*_{i2}U_{j2}+\frac{1}{2}U^*_{i3}U_{j3}+\frac{1}{2}U^*_{i4}U_{j4}-\sin^2\theta_W\delta_{ij} \tag{21}$$

$$O^{L'}_{ij}=-\frac{1}{2}N^*_{i4}N_{j4}+\frac{1}{2}N^*_{i5}N_{j5}-\frac{1}{2}N^*_{i8}N_{j8}+\frac{1}{2}N^*_{i9}N_{j9} \tag{22}$$

$$O^{R'}_{ij}=-O^{L'}_{ij}{}^*. \tag{23}$$

IV. RESULTS AND DISCUSSION

We base our estimates for chargino and neutralino masses and mixing matrices on previous work done on the subject [21]. Both chargino and neutralino masses, as well as their mixing matrices, depend on the following parameters: M_L (we take $M_{B-L}=0.5M_L$), the left-handed gaugino mass parameter, M_R, the right-handed gaugino mass parameter, $|\mu|$ and $\mathrm{sgn}(\mu)$, the Higgsino mass parameter, and $\tan\beta=\kappa_u/\kappa_d$. In addition, the branching ratio will depend on the mass and off-diagonal Yukawa coupling $(h_{LR})_{ij}$ of the $\tilde{\Delta}^{--}_R$: restrictions on the former have appeared before [22], the latter will be taken as light as phenomenologically allowed [14]. (For simplicity, we only consider the contribution of the right-handed doubly charged Higgsino.) The branching ratio also depends on the fractional slepton and sneutrino mass splittings, as well as their mixing. From the study of radiative leptonic decays and $(g-2)_\mu$ these fractional splittings were found to be [3]

$$\delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu}/m^2_{\tilde{\nu}}<1.5\times10^{-5} \tag{24}$$

$$\delta m^2_{\tilde{\nu}_e\tilde{\nu}_\tau}/m^2_{\tilde{\nu}}<4.2\times10^{-2} \tag{25}$$

$$\delta m^2_{\tilde{\nu}_\tau\tilde{\nu}_\mu}/m^2_{\tilde{\nu}}<2.7\times10^{-2} \tag{26}$$

for the sneutrinos, and

$$\delta m^2_{\tilde{e}\tilde{\mu}}/m^2_{\tilde{l}}<6.2\times10^{-3} \tag{27}$$

$$\delta m^2_{\tilde{e}\tilde{\tau}}/m^2_{\tilde{l}}<1.6\times10^{-3} \tag{28}$$

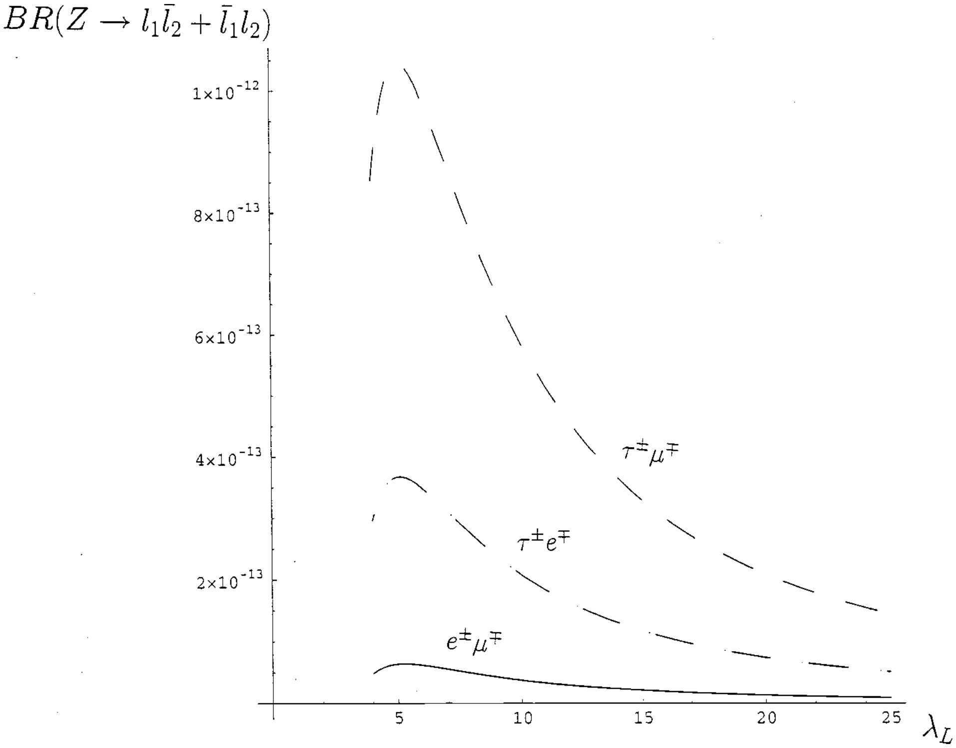

FIG. 1. The branching ratio $BR(Z\to\overline{l_1}l_2+l_1\overline{l_2})$ as a function of the left-handed gaugino mass parameter M_L, for $m_0=100$ GeV, $M_L/\mu\approx1$, $\mu>0$ and $\tan\beta=3$. We take $M_R=10$ TeV and $M_{\tilde{\Delta}^{--}}=120$ GeV. Here $\lambda_L=M_L^2/M_{W_L}^2$. The curves represent: (solid line) $Z\to e^\pm\mu^\mp$ branching ratio; (dot-dashed line) $Z\to e^\pm\tau^\mp$ branching ratio; and (dashed line) $Z\to\mu^\pm\tau^\mp$ branching ratio.

$$\delta m^2_{\tilde{\mu}\tilde{\tau}}/m^2_{\tilde{l}}<10^{-1} \tag{29}$$

for the sleptons. The relative mass splittings are larger than the ones obtained in the MSSM.

We analyze the branching ratio as a function of the left-handed gaugino mass, M_L, and the universal scalar mass m_0, for sgn$(\mu)>0$, $M_L/\mu\approx1$, which are values preferred by the anomalous magnetic moment of the muon. We take $\tan\beta=3$ and $M_R=10$ TeV in all our considerations.

In Fig. 1 we plot the dependence of the branching ratio of $Z\to e\mu$, $e\tau$ and $\mu\tau$, respectively, on the chargino and neutralino mass parameter M_L for fixed universal scalar mass $m_0=100$ GeV. As expected, the branching ratio decreases with increasing $\lambda_L=M_L^2/M_{W_L}^2$ and $BR(Z\to\mu^\pm\tau^\mp)$ could reach at most 2×10^{-12} for low $M_L\approx200$ GeV. The same branching ratio can reach 1.4×10^{-11} for $M_L=100$ GeV and $m_0=150$ GeV, as shown in Fig. 2, where we plot the dependence of the same branching ratios on the universal scalar mass through $\lambda_0=\tilde{m}_0^2/M_{W_L}^2$. Again, the largest contribution

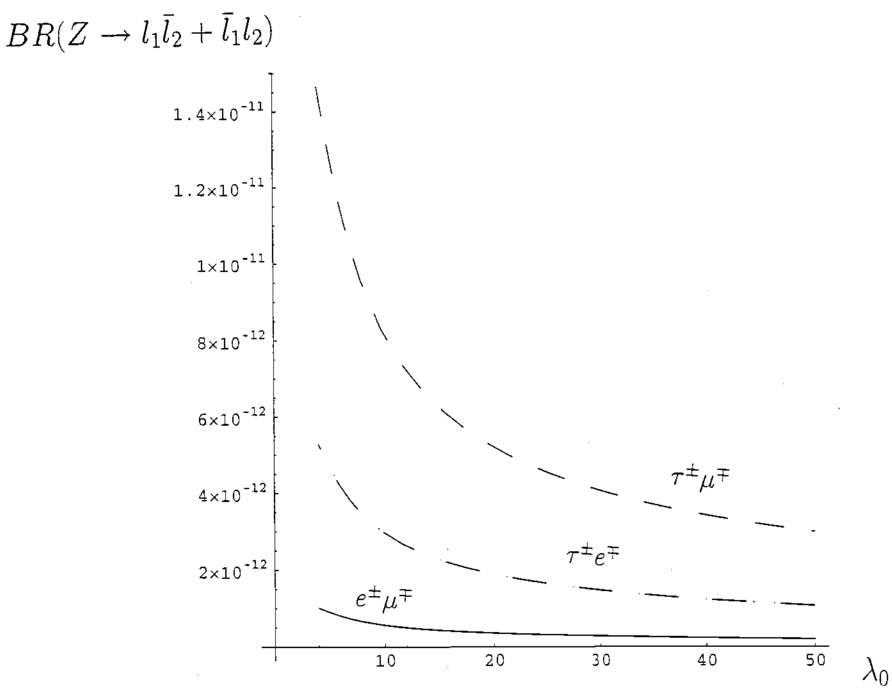

FIG. 2. The branching ratio $BR(Z\to\overline{l_1}l_2+l_1\overline{l_2})$ as a function of the universal scalar mass parameter m_0, for $M_L=100$ GeV, $M_L/\mu\approx1$, $\mu>0$ and $\tan\beta=3$. Here $\lambda_0=\tilde{m}_0^2/M_{W_L}^2$. The curves represent: (solid line) $Z\to e^\pm\mu^\mp$ branching ratio; (dot-dashed line) $Z\to e^\pm\tau^\mp$ branching ratio; and (dashed line) $Z\to\mu^\pm\tau^\mp$ branching ratio.

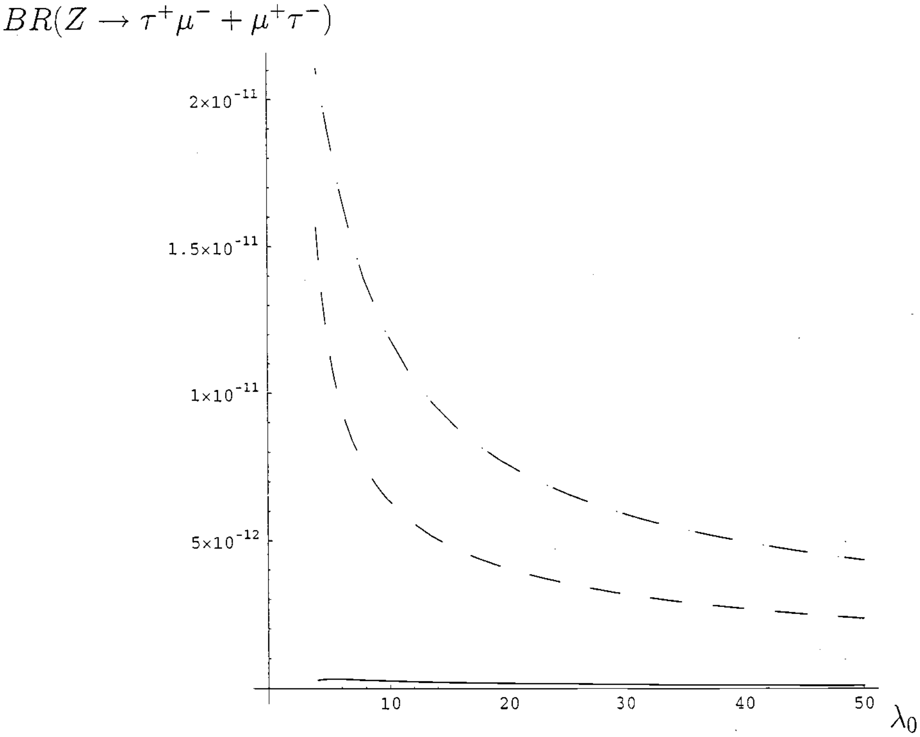

FIG. 3. Relative contributions to the branching ratios of $BR(Z \to \tau^+\mu^- + \mu^+\tau^-)$ from the chargino, neutralino and doubly charged Higgsino components as a function of the universal scalar mass parameter m_0, for $M_L = 100$ GeV, $M_L/\mu \approx 1$, $\mu > 0$ and $\tan\beta = 3$. As before, $\lambda_0 = \tilde{m}_0^2/M_{W_L}^2$. The curves represent: (solid line) 10^3 times the chargino contribution; (dot-dashed line) the neutralino contribution; and (dashed line) 10^3 times the doubly charged Higgsino contribution.

to lepton flavor violating decays of the Z comes from its branching ratio to $\mu\tau$, and that should be true in any model with slepton mixings in agreement with our understanding of neutrino oscillations. Finally in Fig. 3 we plot (for $Z \to \mu^\pm\tau^\mp$) the relative contributions of the chargino, neutralino and doubly charged Higgsino to the branching ratio, as functions of m_0. Some of the general features emerging from the graphs are as follows.

The branching ratio for $Z \to \mu^\pm\tau^\mp$ is the largest of the three lepton flavor violating branching ratios, but still is less than 10^{-11}, and more likely $\mathcal{O}(10^{-12})$ for gaugino mass values favored by the anomalous magnetic moment of the muon. This value is 2–3 orders of magnitude below what could be observed at GigaZ. We expect this ratio to be even smaller in MSSM, where slepton mixings are more restricted than in LRSUSY [2].

In the approximation used, the branching ratios are proportional to the relative slepton mass splittings, such that

$$BR(Z \to \mu^\pm\tau^\mp):BR(Z \to e^\pm\tau^\mp):BR(Z \to e^\pm\mu^\mp) \approx \frac{\delta^2_{\tilde{\mu}\tilde{\tau}}}{\tilde{m}^2}:\frac{\delta^2_{\tilde{e}\tilde{\tau}}}{\tilde{m}^2}:\frac{\delta^2_{\tilde{e}\tilde{\mu}}}{\tilde{m}^2}. \quad (30)$$

As in the case of the dipole transitions, in Z lepton flavor decays the neutralino contribution dominates all processes and the chargino contribution is much smaller. The doubly charged Higgsino contribution is most important for $Z \to \mu^\pm\tau^\mp$ [because of the weaker constraints on $(h_{LR})_{\mu\tau}$], but is still $\mathcal{O}(10^{-3})$ smaller than the neutralino. Thus, the results are not too sensitive to the particular Higgs structure of the model and are more sensitive to its left-right symmetry.

We do not expect the results to be significantly altered by large values of $\tan\beta$. The left-right mass splitting of the sleptons is proportional to $\tan\beta$ and thus expected to increase. This increase will affect both the muon magnetic moment and the lepton flavor violating decays, which will be dominated by slepton-neutralino graphs. The fractional slepton mass, which depends on the ratio of the decay amplitude for $\mu \to e\gamma$ and the anomalous magnetic moment of the muon, will remain fairly constant against variations in $\tan\beta$. In this scenario, although the doubly charged Higgsino contribution will increase, the lepton flavor violating decays of the Z will be dominated by the neutralino-slepton contributions, and will be fairly independent of $\tan\beta$.

In conclusion, we have shown that, including the constraints from radiative lepton decays, lepton flavor violating decays of the Z bosons in the left-right supersymmetric model are expected to be at most in the $10^{-12}-10^{-11}$ range. The prospective high-energy e^+e^- linear collider TESLA is being designed to operate on top of the Z-boson resonance. Given the high luminosity and the cross section, about 2×10^9 Z events could be generated in an operational year of 10^7 s. The signal predicted here is below the experimental sensitivity of the new planned GigaZ experiment at TESLA. A signal of lepton flavor violation in Z decays is not expected in LRSUSY and, if seen, it would be an indication of other models, such as models with heavy Majorana neutrinos [12], or of a new mechanism responsible for the decay [23].

ACKNOWLEDGMENTS

This work was funded by NSERC of Canada (SAP0105354).

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